Microscopic behavior of systems with Coulomb and Riesz interactions

Abstract:

We are interested in systems of points with Coulomb, logarithmic or more generally Riesz interactions (i.e. inverse powers of the distance). They arise in various settings: an instance is the classical Coulomb gas and beta ensembles, another is vortices in the Ginzburg-Landau model of superconductivity, where one observes in certain regimes the emergence of densely packed point vortices forming perfect triangular lattice patterns named Abrikosov lattices, a third is the study of Fekete points which arise in approximation theory. We describe tools to study such systems and derive a next order (beyond mean field limit) "renormalized energy" that governs microscopic patterns of points. We present the derivation of the limiting problem and the question of its minimization and its link with the Abrikosov lattice and crystallization questions. In the statistical mechanics context, we obtain a large deviation principle on the "empirical fields." This is based on joint works with Etienne Sandier, Nicolas Rougerie, Simona Rota Nodari, Mircea Petrache, and Thomas Lebl\'e.