New Hardness Results for Routing on Disjoint Paths

Abstract:

In the classical Node-Disjoint Paths (NDP) problem, the input consists of an undirected n-vertex graph \(G\), and a collection \(M\) of pairs of its vertices, called source-destination, or demand, pairs. The goal is to route the largest possible number of the demand pairs via node-disjoint paths. The best current approximation for the problem is achieved by a simple greedy algorithm, whose approximation factor is \(O(\sqrt{n})\), while the best current negative result is a roughly \(\Omega(\log^{1/2} n)\)-hardness of approximation. Even seemingly simple special cases of the problem are still poorly understood: when the input graph is a grid, the best current algorithm achieves a \(\tilde{O}(n^{1/4})\)- approximation, and when it is a general planar graph, the best current approximation ratio of an efficient algorithm is \(\tilde{O}(n^{9/19})\). The best currently known lower bound for both these versions of the problem is \(\text{APX- hardness}\).

In this talk we will show that NDP is \(2^{\Omega(\log n)}\)-hard to approximate, unless all problems in \(\text{NP}\) have algorithms with running time \(n^{\Omega(\log n)}\). Our result holds even when the underlying graph is a planar graph with maximum vertex degree 3, and all source vertices lie on the boundary of a single face. We extend this result to the closely related Edge-Disjoint Paths problem, showing the same hardness of approximation ratio even for sub-cubic planar graphs with all sources lying on the boundary of a single face.

This is joint work with David H.K. Kim and Rachit Nimavat.