New Hardness Results for Routing on Disjoint Paths

Abstract:

In the classical Node-Disjoint Paths (NDP) problem, the input consists of an undirected n-vertex graph \( G \), and a collection \( M \) of pairs of its vertices, called source-destination, or demand, pairs. The goal is to route the largest possible number of the demand pairs via node-disjoint paths. The best current approximation for the problem is achieved by a simple greedy algorithm, whose approximation factor is \( O(\sqrt{n}) \), while the best current negative result is a roughly \( \Omega(\log^{1/2} n) \)-hardness of approximation. Even seemingly simple special cases of the problem are still poorly understood: when the input graph is a grid, the best current algorithm achieves a \( \tilde{O}(n^{1/4}) \)-approximation, and when it is a general planar graph, the best current approximation ratio of an efficient algorithm is \( \tilde{O}(n^{9/19}) \). The best currently known lower bound for both these versions of the problem is APX-hardness.

In this talk we will show that NDP is \( 2^{\Omega(\log n)} \)-hard to approximate, unless all problems in \( \text{NP} \) have algorithms with running time \( n^{\Omega(\log n)} \). Our result holds even when the underlying graph is a planar graph with maximum vertex degree 3, and all source vertices lie on the boundary of a single face. We extend this result to the closely related Edge-Disjoint Paths problem, showing the same hardness of approximation ratio even for sub-cubic planar graphs with all sources lying on the boundary of a single face.

This is joint work with David H.K. Kim and Rachit Nimavat.