New Results on Learning and Reconstruction of Quantum States

Abstract:

Given an unknown D-dimensional quantum state \( \rho \), as well as \( M \) two-outcome measurements \( E_1, \ldots, E_M \), how many copies of \( \rho \) do we need, if we want to learn the approximate probability that \( E_i \) accepts \( \rho \) for *every* \( i \)? In this talk, I'll prove the surprising result --I didn't believe it myself at first -- that one can achieve this using a number of copies that's polylogarithmic in both \( M \) and \( D \). So, e.g., one can learn whether *every* size-\( n^3 \) quantum circuit accepts or rejects an \( n \)-qubit state, given only poly(\( n \)) copies of the state. To prove this will require first surveying previous results on measuring quantum states and succinctly describing them, including my 2004 postselected learning theorem, and my 2006 "Quantum OR Bound" (with an erroneous proof fixed in 2016 by Harrow, Lin, and Montanaro).

As time permits, I'll also discuss new joint work with Xinyi Chen, Elad Hazan, and Ashwin Nayak, which takes my 2006 result on PAC-learnability of quantum states, and extends to the setting of online learning. Here we show that, given a sequence of \( T \) two-outcome measurements on an \( n \)-qubit state, even if the sequence is chosen adversarially, one can still learn to predict the outcomes of those measurements with total regret \( O(n) \) (in the "realizable" case) or \( O(\sqrt{TN}) \) (in the "non-realizable" case).

No quantum computing background will be assumed.