New tensor categories related to orthogonal and symplectic groups and the strange supergroup $P(\infty)$

Abstract:

We study a symmetric monoidal category of tensor representations of the ind group $O(\infty)$. This category is Koszul and its Koszul dual is the category of tensor representations of the strange supergroup $P(\infty)$. This can be used to compute $Ext$ groups between simple objects in both categories. The above categories are missing the duality functor. It is possible to extend these categories to certain rigid tensor categories satisfying a nice universality property. In the case of $O(\infty)$ such extension depends on a parameter $t$ and is closely related to the Deligne’s category $Rep O(t)$. When $t$ is integer, this new category is a highest weight category and the action of translation functors in this category is related to the representation of $gl(\infty)$ in the Fock space.