New tensor categories related to orthogonal and symplectic groups and the strange supergroup $P(\infty)$

**Abstract:**

We study a symmetric monoidal category of tensor representations of the ind group $O(\infty)$. This category is Koszul and its Koszul dual is the category of tensor representations of the strange supergroup $P(\infty)$. This can be used to compute Ext groups between simple objects in both categories. The above categories are missing the duality functor. It is possible to extend these categories to certain rigid tensor categories satisfying a nice universality property. In the case of $O(\infty)$ such extension depends on a parameter $t$ and is closely related to the Deligne's category $Rep O(t)$. When $t$ is integer, this new category is a highest weight category and the action of translation functors in this category is related to the representation of $gl(\infty)$ in the Fock space.