The nilpotent cone for classical simple Lie superalgebras

Many aspects of the representation theory of a Lie algebra and its associated algebraic group are governed by the geometry of their nilpotent cone. In this talk, we will introduce an analogue of the nilpotent cone $N$ for Lie superalgebras and show that for a simple classical Lie superalgebra the number of nilpotent orbits is finite. We will also show that the commuting variety $X$ described by Duflo and Serganova, which has applications in the study of the finite dimensional representation theory of Lie superalgebras, is contained in $N$. Consequently, the finiteness result on $N$ generalizes and extends the work on the commuting variety. For the general linear Lie superalgebra $\mathfrak{gl}(m|n)$, we will also discuss more detailed geometric results of $N$. In particular, we compute the dimensions of $N$ and the centralizer of a nilpotent orbit, describe the irreducible components of $N$, and show that $N$ is a complete intersection. This is joint work with Daniel Nakano from the University of Georgia.