Nilpotent orbits associated to distinguished representations of reductive groups.

Abstract:

Let $G$ be a reductive group over a local field $F$ of characteristic zero, and $H$ be a spherical subgroup. An irreducible representation of $G$ is said to be distinguished by $H$ if it has an $H$-invariant continuous linear functional. The study of distinguished representations is of much current interest, because of their relation to the Plancherel measure on $G/H$ and to periods of automorphic forms.

While a complete classification seems to be out of reach, we established simple micro-local necessary conditions for distinction. The conditions are formulated in terms of the nilpotent orbits associated to the representation, in the spirit of the orbit method. Our results are strongest for Archimedean $F$. In this case, Rossmann showed that for any irreducible Casselman-Wallach representation, the Zariski closure of the wave-front set is the closure of a unique nilpotent complex orbit. We have shown that the restriction of this orbit to the complexified Lie algebra of $H$ includes zero.

We apply this result to symmetric pairs, branching problems, and parabolic induction. We also have a twisted version for the case when $\chi$ has a functional invariant with respect to an "additive" character of $H$. As an application of our theorem we derive necessary conditions for the existence of Rankin-Selberg, Bessel, Klyachko and Shalika models. Our results are compatible with the recent Gan-Gross-Prasad conjectures for non-generic representations. Our necessary conditions are rarely sufficient, but they are sufficient for one class of models: the Klyachko models for unitary representations of general linear groups.

This is a joint work with Eitan Sayag.