Nodal statistics of graph eigenfunctions

Abstract:

Understanding statistical properties of zeros of Laplacian eigenfunctions is a program which is attracting much attention from mathematicians and physicists. We will discuss this program in the setting of "quantum graphs", self-adjoint differential operators acting on functions living on a metric graph. Numerical studies of quantum graphs motivated a conjecture that the distribution of nodal surplus (a suitably rescaled number of zeros of the n-th eigenfunction) has a universal form: it approaches Gaussian as the number of cycles grows. The first step towards proving this conjecture is a result established for graphs which are composed of cycles separated by bridges. For such graphs we use the nodal-magnetic theorem of the speaker, Colin de Verdiere and Weyand to prove that the distribution of the nodal surplus is binomial with parameters $p=1/2$ and $n$ equal to the number of cycles. Based on joint work with Lior Alon and Ram Band.