Non-commutative Iwasawa algebras are completed group rings of compact p-adic Lie groups with mod-p, or p-adic integer, coefficients. They can also be viewed as rings of continuous p-adic distributions on the group in question. These algebras have found applications in several areas of number theory, including non-commutative Iwasawa theory and the p-adic local Langlands correspondence, but they also provide interesting examples of non-commutative Noetherian rings which are similar in certain respects to universal enveloping algebras of finite dimensional Lie algebras. After giving the basic definitions and some examples, I will advertise some open questions on the algebraic structure of these Iwasawa algebras.