Optimal transport and geodesics on diffeomorphism groups

Abstract:

We revisit how the Euler and Burgers equations arise as geodesics on the groups of diffeomorphisms. It turns out that the Euler hydrodynamics is in a sense dual to problems of optimal mass transport. We also describe L^2 and H^1 versions of the the Wasserstein space of volume forms. It turns out that for the homogeneous H^1 metric the Wasserstein space is isometric to (a piece of) an infinite-dimensional sphere and it leads to an integrable generalization of the Hunter-Saxton equation.