Packing Small Vectors

Abstract:

Online d-dimensional vector packing models many settings such as minimizing resources in data centers where jobs have multiple resource requirements (CPU, Memory, etc.). However, no online d-dimensional vector packing algorithm can achieve a competitive ratio better than d. Fortunately, in many natural applications, vectors are relatively small, and thus the lower bound does not hold. For sufficiently small vectors, an $O(\log d)$-competitive algorithm was known. We improve this to a constant competitive ratio, arbitrarily close to $e$ (where $e$ is the base of the natural logarithm), given that vectors are sufficiently small.

We give improved results for the two dimensional case. For arbitrarily small vectors, the First Fit algorithm for two dimensional vector packing is no better than 2-competitive. We present a natural family of First Fit variants, and for optimized parameters get a competitive ratio of approximately 1.48 for sufficiently small vectors.

We improve upon the 1.48 competitive ratio - not via a First Fit variant - and give a competitive ratio arbitrarily close to $4/3$ for packing small, two dimensional vectors. We show that no algorithm can achieve better than a $4/3$ competitive ratio for two dimensional vectors, even if one allows the algorithm to split vectors among arbitrarily many bins.