Parallel Repetition From Fortification

The Parallel Repetition Theorem upper-bounds the value of a repeated (tensored) two prover game in terms of the value of the base game and the number of repetitions. In this work we give a simple transformation on games -- "fortification" -- and show that for fortified games, the value of the repeated game decreases perfectly exponentially with the number of repetitions, up to an arbitrarily small additive error. Our proof is combinatorial and (very) short. As corollaries, we obtain: (1) Starting from a PCP Theorem with soundness error bounded away from 1, we get a PCP with arbitrarily small constant soundness error. In particular, starting with the combinatorial PCP of Dinur, we get a combinatorial PCP with low error. The latter can be used for hardness of approximation as in the work of Hastad. (2) Starting from the work of the author and Raz, we get a projection PCP theorem with the smallest soundness error known today. The theorem yields nearly a quadratic improvement in the size compared to previous work.