Consider a real Gaussian stationary process, either on $\mathbb{Z}$ or on $\mathbb{R}$. What is the probability that it remains positive on $[0,N]$ for large $N$?

The relation between this probability, known as the persistence probability, and the covariance kernel of the process has been investigated since the 1950s with motivations stemming from probability, engineering and mathematical physics. Nonetheless, until recently, good estimates were known only for particular cases, or when the covariance kernel is either non-negative or summable.

In the first hour of the talk we will discuss new spectral methods which greatly simplify the analysis of persistence. We will then describe its qualitative behavior in a very general setting.

In the second hour, we will describe (very) recent progress. In particular we will show the proof of the "spectral gap conjecture", which states: if the spectral measure vanishes on an interval containing 0 then the persistence is less than $e^{-cN^2}$, and this bound is tight if the measure is non-singular and compactly supported.

Time permitting, we will also discuss "tiny persistence" phenomena (of the order of $e^{-e^{cN}}$).

Based on joint works with Ohad Feldheim, Benjamin Jaye, Fedor Nazarov and Shahaf Nitzan.