On the phase transition in random simplicial complexes

Abstract:

It is well-known that the model of random graphs undergoes a dramatic change around \( p = \frac{1}{n} \). It is here that the random graph is, almost surely, no longer a forest, and here it first acquires a giant connected component. Several years ago, Linial and Meshulam have introduced the \( X_d(n,p) \) model, a probability space of \( n \)-vertex \( d \)-dimensional simplicial complexes, where \( X_1(n,p) \) coincides with \( G(n,p) \). Within this model we prove a natural \( d \)-dimensional analog of these graph theoretic phenomena. Specifically, we determine the exact threshold for the nonvanishing of the real \( d \)-th homology of complexes from \( X_d(n,p) \), and show that it is strictly greater than the threshold of \( d \)-collapsibility. In addition, we compute the real Betti numbers, i.e. the dimension of the homology groups, of \( X_d(n,p) \) for \( p = c/n \). Finally, we establish the emergence of giant shadow at this threshold. (For \( d = 1 \) a giant shadow and a giant component are equivalent). Unlike the case for graphs, for \( d > 1 \) the emergence of the giant shadow is a first order phase transition. The talk will contain the necessary topological background on simplicial complexes, and will focus on the main idea of the proof: the local weak limit of random simplicial complexes and its role in the analysis of phase transitions. Joint work with Nati Linial.