



THE WEIZMANN INSTITUTE OF SCIENCE  
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Geometric Functional Analysis and Probability Seminar

Room 261 ,Ziskind Building  
on Thursday, Feb 05, 2015  
at 11:05

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On the phase transition in random simplicial complexes

Abstract:

It is well-known that the model of random graphs undergoes a dramatic change around  $p=1/n$ . It is here that the random graph is, almost surely, no longer a forest, and here it first acquires a giant connected component. Several years ago, Linial and Meshulam have introduced the  $X_d(n,p)$  model, a probability space of  $n$ -vertex  $d$ -dimensional simplicial complexes, where  $X_1(n,p)$  coincides with  $G(n,p)$ . Within this model we prove a natural  $d$ -dimensional analog of these graph theoretic phenomena. Specifically, we determine the exact threshold for the nonvanishing of the real  $d$ -th homology of complexes from  $X_d(n,p)$ , and show that it is strictly greater than the threshold of  $d$ -collapsibility. In addition, we compute the real Betti numbers, i.e. the dimension of the homology groups, of  $X_d(n,p)$  for  $p=c/n$ . Finally, we establish the emergence of giant shadow at this threshold. (For  $d=1$  a giant shadow and a giant component are equivalent). Unlike the case for graphs, for  $d > 1$  the emergence of the giant shadow is a first order phase transition. The talk will contain the necessary topological background on simplicial complexes, and will focus on the main idea of the proof: the local weak limit of random simplicial complexes and its role in the analysis of phase transitions. Joint work with Nati Linial.