Point counting for foliations over number fields

Abstract:

We consider an algebraic variety $V$ and its foliation, both defined over a number field. Given a (compact piece of a) leaf $L$ of the foliation, and a subvariety $W$ of complementary codimension, we give an upper bound for the number of intersections between $L$ and $W$. The bound depends polynomially on the degree of $W$, the logarithmic height of $W$, and the logarithmic distance between $L$ and the locus of points where leafs of the foliation intersect $W$ improperly. Using this theory we prove the Wilkie conjecture for sets defined using leafs of foliations under a certain assumption about the algebraicity locus. For example, we prove the if none of the leafs contain algebraic curves then the number of algebraic points of degree $d$ and log-height $h$ on a (compact piece of a) leaf grows polynomially with $d$ and $h$. This statement and its generalizations have many applications in diophantine geometry following the Pila-Zannier strategy.

I will focus mostly on the proof of the main statement, which uses a combination of differential-algebraic methods related to foliations with some ideas from complex geometry and value distribution theory. If time permits I will briefly discuss the applications to counting algebraic points and diophantine geometry at the end.