Prahladh Harsha
Tata Institute of Fundamental Research and Weizmann Institute

On polynomial approximations to $\text{AC}^0$

Abstract:

In this talk, we will discuss some questions related to polynomial approximations of $\text{AC}^0$. A classic result due to Tarui (1991) and Beigel, Reingold, and Spielman (1991), states that any $\text{AC}^0$ circuit of size $s$ and depth $d$ has an $\epsilon$-error probabilistic polynomial over the reals of degree at most $(\log(s/\epsilon))^\mathrm{O}(d)$. We will have a re-look at this construction and show how to improve the bound to $(\log s)^\mathrm{O}(d) \cdot \log(1/\epsilon)$, which is much better for small values of $\epsilon$. As an application of this result, we show that $(\log s)^\mathrm{O}(d) \cdot \log(1/\epsilon)$-wise independence fools $\text{AC}^0$, improving on Tal's strengthening of Braverman's theorem that $(\log(s/\epsilon))^\mathrm{O}(d)$-wise independence fools $\text{AC}^0$. Time permitting, we will also discuss some lower bounds on the best polynomial approximations to $\text{AC}^0$.

Joint work with Srikanth Srinivasan.