The Power of Distributed Verifiers in Interactive Proofs

Abstract:

We explore the power of interactive proofs with a distributed verifier. In this setting, the verifier consists of \( n \) nodes and a graph \( G \) that defines their communication pattern. The prover is a single entity that communicates with all nodes by short messages. The goal is to verify that the graph \( G \) belongs to some language in a small number of rounds, and with small communication bound, i.e., the proof size.

This interactive model was introduced by Kol, Oshman and Saxena (PODC 2018) as a generalization of non-interactive distributed proofs. They demonstrated the power of interaction in this setting by constructing protocols for problems as Graph Symmetry and Graph Non-Isomorphism -- both of which require proofs of \( (n^2) \)-bits without interaction.

In this work, we provide a new general framework for distributed interactive proofs that allows one to translate standard interactive protocols (i.e., with a centralized verifier) to ones where the verifier is distributed with a proof size that depends on the computational complexity of the verification algorithm run by the centralized verifier. We show the following:

* Every (centralized) computation performed in time \( O(n) \) on a RAM can be translated into three-round distributed interactive protocol with \( O(\log n) \) proof size. This implies that many graph problems for sparse graphs have succinct proofs (e.g., testing planarity).

* Every (centralized) computation implemented by either a small space or by uniform NC circuit can be translated into a distributed protocol with \( O(1) \) rounds and \( O(\log n) \) bits proof size for the low space case and \( \text{polylog}(n) \) many rounds and proof size for NC.

* We show that for Graph Non-Isomorphism, one of the striking demonstrations of the power of interaction, there is a 4-round protocol with \( O(\log n) \) proof size, improving upon the \( O(n^4 \log n) \) proof.
size of Kol et al.

* For many problems, we show how to reduce proof size below the seemingly natural barrier of log n. By employing our RAM compiler, we get a 5-round protocol with proof size $O(loglog n)$ for a family of problems including Fixed Automorphism, Clique and Leader Election (for the latter two problems we actually get $O(1)$ proof size).

* Finally, we discuss how to make these proofs non-interactive \emph{arguments} via random oracles.

Our compilers capture many natural problems and demonstrate the difficulty in showing lower bounds in these regimes.
Joint work with Moni Naor and Merav Parter.