The Power of Distributed Verifiers in Interactive Proofs

Abstract:

We explore the power of interactive proofs with a distributed verifier. In this setting, the verifier consists of \( n \) nodes and a graph \( G \) that defines their communication pattern. The prover is a single entity that communicates with all nodes by short messages. The goal is to verify that the graph \( G \) belongs to some language in a small number of rounds, and with small communication bound, i.e., the proof size.

This interactive model was introduced by Kol, Oshman and Saxena (PODC 2018) as a generalization of non-interactive distributed proofs. They demonstrated the power of interaction in this setting by constructing protocols for problems as Graph Symmetry and Graph Non-Isomorphism -- both of which require proofs of \( (n^2) \)-bits without interaction.

In this work, we provide a new general framework for distributed interactive proofs that allows one to translate standard interactive protocols (i.e., with a centralized verifier) to ones where the verifier is distributed with a proof size that depends on the computational complexity of the verification algorithm run by the centralized verifier. We show the following:

* Every (centralized) computation performed in time \( O(n) \) on a RAM can be translated into three-round distributed interactive protocol with \( O(\log n) \) proof size. This implies that many graph problems for sparse graphs have succinct proofs (e.g., testing planarity).

* Every (centralized) computation implemented by either a small space or by uniform NC circuit can be translated into a distributed protocol with \( O(1) \) rounds and \( O(\log n) \) bits proof size for the low space case and polylog\( (n) \) many rounds and proof size for NC.

* We show that for Graph Non-Isomorphism, one of the striking demonstrations of the power of interaction, there is a 4-round protocol with \( O(\log n) \) proof size, improving upon the \( O(n^{\log n}) \) proof
size of Kol et al.

* For many problems, we show how to reduce proof size below the seemingly natural barrier of log n. By employing our RAM compiler, we get a 5-round protocol with proof size $O(\log\log n)$ for a family of problems including Fixed Automorphism, Clique and Leader Election (for the latter two problems we actually get $O(1)$ proof size).

* Finally, we discuss how to make these proofs non-interactive \emph{arguments} via random oracles.

Our compilers capture many natural problems and demonstrate the difficulty in showing lower bounds in these regimes.
Joint work with Moni Naor and Merav Parter.