Abstract:

We explore the power of interactive proofs with a distributed verifier. In this setting, the verifier consists of $n$ nodes and a graph $G$ that defines their communication pattern. The prover is a single entity that communicates with all nodes by short messages. The goal is to verify that the graph $G$ belongs to some language in a small number of rounds, and with small communication bound, i.e., the proof size.

This interactive model was introduced by Kol, Oshman and Saxena (PODC 2018) as a generalization of non-interactive distributed proofs. They demonstrated the power of interaction in this setting by constructing protocols for problems as Graph Symmetry and Graph Non-Isomorphism -- both of which require proofs of $(n^2)$-bits without interaction.

In this work, we provide a new general framework for distributed interactive proofs that allows one to translate standard interactive protocols (i.e., with a centralized verifier) to ones where the verifier is distributed with a proof size that depends on the computational complexity of the verification algorithm run by the centralized verifier. We show the following:

* Every (centralized) computation performed in time $O(n)$ on a RAM can be translated into three-round distributed interactive protocol with $O(\log n)$ proof size. This implies that many graph problems for sparse graphs have succinct proofs (e.g., testing planarity).

* Every (centralized) computation implemented by either a small space or by uniform NC circuit can be translated into a distributed protocol with $O(1)$ rounds and $O(\log n)$ bits proof size for the low space case and polylog($n$) many rounds and proof size for NC.

* We show that for Graph Non-Isomorphism, one of the striking demonstrations of the power of interaction, there is a 4-round protocol with $O(\log n)$ proof size, improving upon the $O(n^*\log n)$ proof size.
size of Kol et al.

* For many problems, we show how to reduce proof size below the seemingly natural barrier of \( \log n \). By employing our RAM compiler, we get a 5-round protocol with proof size \( O(\log \log n) \) for a family of problems including Fixed Automorphism, Clique and Leader Election (for the latter two problems we actually get \( O(1) \) proof size).

* Finally, we discuss how to make these proofs non-interactive \( \text{arguments} \) via random oracles.

Our compilers capture many natural problems and demonstrate the difficulty in showing lower bounds in these regimes.

Joint work with Moni Naor and Merav Parter.