Quantum affine algebras and Grassmannians

Abstract:

Let $\mathfrak{g} = \mathfrak{sl}_n$ and $U_q(\widehat{\mathfrak{g}})$ the corresponding quantum affine algebra. Hernandez and Leclerc proved that there is an isomorphism $\Phi$ from the Grothendieck ring $\mathcal{R}_{\ell}$ of a certain subcategory $\mathcal{C}_{\ell}$ of finite-dimensional $U_q(\widehat{\mathfrak{g}})$-modules to a certain quotient $\mathbb{C}[\text{Gr}(n, n+\ell+1, \sim)]$ of a Grassmannian cluster algebra. We proved that this isomorphism induces an isomorphism $\tilde{\Phi}$ from the monoid of dominant monomials to the monoid of semi-standard Young tableaux. Using this result and the results of Qin and the results of Kashiwara, Kim, Oh, and Park, we have that every cluster monomial (resp. cluster variable) in a Grassmannian cluster algebra is of the form $\text{ch}(T)$ for some real (resp. prime real) rectangular semi-standard Young tableau $T$, where $\text{ch}(T)$ is certain map obtained from a formula of Arakawa--Suzuki. We also translated Arakawa--Suzuki's formula to the setting of $q$-characters and apply it to study real modules, prime modules, and compatibility of cluster variables.

This is joint work with Wen Chang, Bing Duan, and Chris Fraser.