Random Cayley graphs

Abstract:

We consider the random Cayley graph of a finite group $G$ formed by picking $k$ random generators uniformly at random:

1. We prove universality of cutoff (for the random walk) and a concentration of measure phenomenon in the Abelian setup (namely, that all but $o(|G|)$ elements lie at distance $[R-o(R),R-o(R)]$ from the origin, where $R$ is the minimal ball in $Z^k$ of size at least $|G|$), provided $k \gg 1$ is large in terms of the size of the smallest generating set of $G$. As conjectured by Aldous and Diaconis, the cutoff time is independent of the algebraic structure (it is given by the time at which the entropy of a random walk on $Z^k$ is $\log|G|$).
2. We prove analogous results for the Heisenberg $H_{p,d}$ groups of $d \times d$ uni-upper triangular matrices with entries defined mod $p$, for $p$ prime and $d$ fixed or diverging slowly.
3. Lastly, we resolve a conjecture of D. Wilson that if $G$ is a group of size at most $2^d$ then for all $k$ its mixing time in this model is as rapid as that of $Z_2^d$ and likewise, that the slowest mixing $p$-group of a given size is $Z_p^d$.

(Joint work with Sam Thomas.)