



THE WEIZMANN INSTITUTE OF SCIENCE
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE
Geometric Functional Analysis and Probability Seminar

Room 155 ,Ziskind Building
on Thursday, Aug 16, 2018
at 13:30

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Random graphs with constant r -balls

Abstract:

Let F be a fixed infinite, vertex-transitive graph. We say a graph G is r -locally F if for every vertex v of G , the ball of radius r and centre v in G is isometric to the ball of radius r in F . For each positive integer n , let G_n be a graph chosen uniformly at random from the set of all unlabelled, n -vertex graphs that are r -locally F . We investigate the properties that the random graph G_n has with high probability --- i.e., how these properties depend on the fixed graph F .

We show that if F is a Cayley graph of a torsion-free group of polynomial growth, then there exists a positive integer r_0 such that for every integer r at least r_0 , with high probability the random graph $G_n = G_n(F,r)$ defined above has largest component of size between n^{c_1} and n^{c_2} , where $0 < c_1 < c_2 < 1$ are constants depending upon F alone, and moreover that G_n has a rather large automorphism group. This contrasts sharply with the random d -regular graph $G_n(d)$ (which corresponds to the case where F is replaced by the infinite d -regular tree).

Our proofs use a mixture of results and techniques from group theory, geometry and combinatorics.

We obtain somewhat more precise results in the case where F is L^d (the standard Cayley graph of Z^d): for example, we obtain quite precise estimates on the number of n -vertex graphs that are r -locally L^d , for r at least linear in d .

Many intriguing open problems remain: concerning groups with torsion, groups with faster than polynomial growth, and what happens for more general structures than graphs.

This is joint work with Itai Benjamini (WIS).