Abstract:

Let \( F \) be a fixed infinite, vertex-transitive graph. We say a graph \( G \) is `r-locally \( F \)` if for every vertex \( v \) of \( G \), the ball of radius \( r \) and centre \( v \) in \( G \) is isometric to the ball of radius \( r \) in \( F \). For each positive integer \( n \), let \( G_n \) be a graph chosen uniformly at random from the set of all unlabelled, \( n \)-vertex graphs that are \( r \)-locally \( F \). We investigate the properties that the random graph \( G_n \) has with high probability --- i.e., how these properties depend on the fixed graph \( F \).

We show that if \( F \) is a Cayley graph of a torsion-free group of polynomial growth, then there exists a positive integer \( r_0 \) such that for every integer \( r \) at least \( r_0 \), with high probability the random graph \( G_n = G_n(F,r) \) defined above has largest component of size between \( n^{c_1} \) and \( n^{c_2} \), where \( 0 < c_1 < c_2 < 1 \) are constants depending upon \( F \) alone, and moreover that \( G_n \) has a rather large automorphism group. This contrasts sharply with the random \( d \)-regular graph \( G_n(d) \) (which corresponds to the case where \( F \) is replaced by the infinite \( d \)-regular tree).

Our proofs use a mixture of results and techniques from group theory, geometry and combinatorics.

We obtain somewhat more precise results in the case where \( F \) is \( L^d \) (the standard Cayley graph of \( Z^d \)): for example, we obtain quite precise estimates on the number of \( n \)-vertex graphs that are \( r \)-locally \( L^d \), for \( r \) at least linear in \( d \).

Many intriguing open problems remain: concerning groups with torsion, groups with faster than polynomial growth, and what happens for more general structures than graphs.

This is joint work with Itai Benjamini (WIS).