Random walks on planar graphs

Abstract:

We will discuss several results relating the behavior of a random walk on a planar graph and the geometric properties of a nice embedding of the graph in the plane (specifically, circle packing of the graph). One such a result is that for a bounded degree graph, the simple random walk is recurrent if and only if the boundary of the nice embedding is a polar set (that is, Brownian motion misses it almost surely). If the degrees are unbounded, this is no longer true, but for the case of circle packing of a triangulation, there are weights which are obtained naturally from the circle packing, such that when the boundary is polar, the weighted random walk is recurrent (we believe the converse also hold). These weights arise also in the context of discrete holomorphic and harmonic functions, a discrete analog of complex holomorphic functions. We show that as the sizes of circles, or more generally, the lengths of edges in the nice embedding of the graph tend to zero, the discrete harmonic functions converge to their continuous counterpart with the same boundary conditions. Equivalently, that the exit measure of the weighted random walk converges to the exit measure of standard Brownian motion. This improves previous results of Skopenkov 2013 and Werness 2015, who proves similar results under additional local and global assumptions on the embedding. In particular, we make no assumptions on the degrees of the graph, making the result applicable to models of random planar maps.

Based on joint works with Daniel Jerison, Asaf Nachmias, Matan Seidel and Juan Souto.