Randomized Online Matching in Regular Graphs

Abstract:

We study the classic bipartite matching problem in the online setting, first introduced in the seminal work of Karp, Vazirani and Vazirani. Specifically, we consider the problem for the well-studied class of regular graphs. Matching in this class of graphs was studied extensively in the offline setting. In the online setting, an optimal deterministic algorithm, as well as efficient algorithms under stochastic input assumptions were known. In this work, we present a novel randomized algorithm with competitive ratio tending to one on this family of graphs, under adversarial arrival order. Our main contribution is a novel algorithm which achieves competitive ratio \(1 - O(\sqrt{\log d}/\sqrt{d})\) in expectation on \(d\)-regular graphs. In contrast, we show that all previously-known online algorithms, such as the generally worst-case optimal ranking algorithm of Karp et al., are restricted to a competitive ratio strictly bounded away from one, even as \(d\) grows. Moreover, we show the convergence rate of our algorithm’s competitive ratio to one is nearly tight, as no algorithm achieves competitive ratio better than \(1 - O(1/\sqrt{d})\). Finally, we show that our algorithm yields a similar competitive ratio with high probability, as well as guaranteeing each offline vertex a probability of being matched tending to one.