On Recent Advances of the 3D Euler Equations by Means of Examples

Abstract:

In this talk we will use a basic example of shear flow to demonstrate some of the recent advances in the three-dimensional Euler equations. Specifically, this example was introduced by DiPerna and Majda to show that weak limit of classical solutions of Euler equations may, in some cases, fail to be a weak solution of Euler equations. We use this shear flow example to show the immediate loss of smoothness and ill-posedness of solutions of the 3D Euler equations, for initial data that do not belong to $C^{1,\alpha}$. Moreover, we also show the existence of weak solutions for the 3D Euler equations with vorticity that is having a nontrivial density concentrated on non-smooth surface (vortex sheet). This is very different from what has been proven for the two-dimensional Kelvin-Helmholtz (Birkhoff-Rott) problem where a minimal regularity implies the real analyticity of the interface. Furthermore, we use this shear flow to provide explicit examples of non-regular solutions of the three-dimensional Euler equations that conserve the energy, an issue which is related to the Onsager conjecture. Eventually, we will discuss the recent remarkable work of De Lellis and Székelyhidi concerning the wild weak solutions of Euler equations and their non-uniqueness. In particular, we propose the following ruling out criterion for non-physical weak solutions of Euler equations: “in the absence of physical boundaries any weak solution of Euler equations which is not a vanishing viscosity limit of Leray-Hopf weak solutions of the Navier-Stokes equations should be ruled out”. We will use this shear flow, and other solutions of Euler equations with certain spatial symmetry, to provide nontrivial examples for the use of this ruling out criterion. If time allows we will also discuss (i) recent progress concerning the Onsager conjecture in bounded domains; (ii) the nonuniqueness of weak solutions to the 3D Navier-Stokes equations with Hyper-viscosity $(-\Delta)^\theta$, for $\theta < 5/4$, demonstrating the sharpness of the J.-L. Lions result.

This is a joint work with Claude Bardos.