Reciprocity laws and K-theory

Abstract:

We associate to a full flag $F$ in an $n$-dimensional variety $X$ over a field $k$, a "symbol map" $\mu_F : K(F_X) \to \Sigma^n K(k)$. Here, $F_X$ is the field of rational functions on $X$, and $K(.)$ is the $K$-theory spectrum.

We prove a "reciprocity law" for these symbols: Given a partial flag, the sum of all symbols of full flags refining it is $0$. Examining this result on the level of $K$-groups, we derive the following known reciprocity laws: the degree of a principal divisor is zero, the Weil reciprocity law, the residue theorem, the Contou-Carrère reciprocity law (when $X$ is a smooth complete curve) as well as the Parshin reciprocity law and the higher residue reciprocity law (when $X$ is higher-dimensional).

This is a joint work with Evgeny Musicantov.