A relative de Rham theorem for Nash Submersions

Abstract:

For a Nash manifold X and a Nash vector bundle E on X, one can form the topological vector space of Schwartz sections of E, i.e. the smooth sections which decay fast along with all derivatives. It was shown by Aizenbud and Gourevitch, and independently by Luca Prelli, that for a Nash manifold X, the complex of Schwartz sections of the de Rham complex of X has cohomologies isomorphic to the compactly supported cohomologies of X.

In my talk I will present a work in progress, joint with Avraham Aizenbud, to generalize this result to the relative case, replacing the Nash manifold M with a Nash submersion f: M --> N. Using infinity categorical methods, I will define the notion of a Schwartz section of a Nash bundle E over a complex of sheaves with constructible cohomologies, generalizing the notion of Schwartz section on an open semialgebraic set. I will then relate the Schwartz sections of the relative de Rham complex of a Nash submersion f: M --> N with the Schwartz functions on N over the derived push-forward with proper support of the constant sheaf on M. Finally, I will conclude with some applications to the relation between the Schwartz sections of the relative de Rham complex and the topology of the fibers of f.

Zoom meeting: https://weizmann.zoom.us/j/98304397425