Robust sensitivity

Abstract:

The sensitivity conjecture is a famous open problem in the theory of boolean functions. Let $f$ be a boolean function defined on the hypercube. The sensitivity of a node $x$ is the number of its neighbours in the hypercube, for which $f$ give the opposite value as that it does on $x$. The sensitivity conjecture speculates that if all nodes have low sensitivity, then the function $f$ must be simple. Concretely, all its Fourier mass is supported on levels with low hamming weight.

Recently, Gopalan et al [CCC 2016] conjectured a robust analogue of the sensitivity conjecture: if most of the nodes have low sensitivity, then most of the Fourier mass is supported on levels with low hamming weight. They also prove it under the stronger assumption that all nodes have low sensitivity. In this work, we prove this conjecture, with near tight quantitative bounds.

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