The role of Puiseux characteristics in the local Poincaré problem

Abstract:

The usual Poincaré problem consists in determining whether or not there exist upper bounds for the degree of an invariant algebraic curve of a foliation of the complex projective plane in terms of the degree of the foliation. We are interested in the local analogue, in which degrees are replaced with vanishing multiplicities at the origin. Since cusps of arbitrary degree can be invariant curves of foliations of degree one in the global setting and of multiplicity one in the local one, there are no universal upper bounds. In order to deal with this issue, extra hypotheses were considered, either on the invariant curve (Cerveau - Lins Neto,...) or on the desingularization of the foliation (Carnicer...).

We consider an irreducible germ of invariant curve and no extra hypotheses. We provide a lower bound for the multiplicity of a germ of complex foliation in dimension 2 in terms of Puiseux characteristics of an irreducible invariant curve. Obviously, the lower bound is trivial for a cusp but it is valuable if the curve has at least two Puiseux characteristic exponents. This is a joint work with Jose Cano and Pedro Fortuny.