Scenery Reconstruction for a Random Walk on Random Scenery with Adversarial Error Insertion

Abstract:
Consider a simple random walk on $\mathbb{Z}$ with a random coloring of $\mathbb{Z}$. Look at the sequence of the first $N$ steps taken and colors of the visited locations. From it, you can deduce the coloring of approximately $\sqrt{N}$ integers. Suppose an adversary may change $\delta N$ entries in that sequence. What can be deduced now? We show that for any $\theta<0.5, p>0$, there are $N_0, \delta_0$ such that if $N>N_0$ and $\delta<\delta_0$ then with probability $>1-p$ we can reconstruct the coloring of $>N^\theta$ integers.