Scenery Reconstruction for a Random Walk on Random Scenery with Adversarial Error Insertion

Abstract:
Consider a simple random walk on \(\mathbb{Z}\) with a random coloring of \(\mathbb{Z}\). Look at the sequence of the first \(N\) steps taken and colors of the visited locations. From it, you can deduce the coloring of approximately \(\sqrt{N}\) integers. Suppose an adversary may change \(\delta N\) entries in that sequence. What can be deduced now? We show that for any \(\theta < 0.5, p > 0\), there are \(N_0, \delta_0\) such that if \(N > N_0\) and \(\delta < \delta_0\) then with probability \(> 1 - p\), we can reconstruct the coloring of \(> N^{\theta}\) integers.