Semisimplification of tensor categories

Abstract:

We develop the theory of semisimplifications of tensor categories defined by Barrett and Westbury. By definition, the semisimplification of a tensor category is its quotient by the tensor ideal of negligible morphisms, i.e., morphisms \( f \) such that \( \text{Tr}(fg)=0 \) for any morphism \( g \) in the opposite direction. In particular, we compute the semisimplification of the category of representations of a finite group in characteristic \( p \) in terms of representations of the normalizer of its Sylow \( p \)-subgroup. This allows us to compute the semisimplification of the representation category of the symmetric group \( S_{n+p} \) in characteristic \( p \), where \( n=0,\ldots,p-1 \), and of the Deligne category \( \text{Rep}^{\{ab\}} S_t \), \( t \in \mathbb{N} \). We also compute the semisimplification of the category of representations of the Kac-De Concini quantum group of the Borel subalgebra of \( \text{sl}_2 \). Finally, we study tensor functors between Verlinde categories of semisimple algebraic groups arising from the semisimplification construction, and objects of finite type in categories of modular representations of finite groups (i.e., objects generating a fusion category in the semisimplification).

This is joint work with Victor Ostrik.