Signal Modeling: From Convolutional Sparse Coding to Convolutional Neural Networks

Abstract:
Within the wide field of sparse approximation, convolutional sparse coding (CSC) has gained increasing attention in recent years. This model assumes a structured-dictionary built as a union of banded Circulant matrices. Most attention has been devoted to the practical side of CSC, proposing efficient algorithms for the pursuit problem, and identifying applications that benefit from this model. Interestingly, a systematic theoretical understanding of CSC seems to have been left aside, with the assumption that the existing classical results are sufficient.

In this talk we start by presenting a novel analysis of the CSC model and its associated pursuit. Our study is based on the observation that while being global, this model can be characterized and analyzed locally. We show that uniqueness of the representation, its stability with respect to noise, and successful greedy or convex recovery are all guaranteed assuming that the underlying representation is locally sparse. These new results are much stronger and informative, compared to those obtained by deploying the classical sparse theory.

Armed with these new insights, we proceed by proposing a multi-layer extension of this model, ML-CSC, in which signals are assumed to emerge from a cascade of CSC layers. This, in turn, is shown to be tightly connected to Convolutional Neural Networks (CNN), so much so that the forward-pass of the CNN is in fact the Thresholding pursuit serving the ML-CSC model. This connection brings a fresh view to CNN, as we are able to attribute to this architecture theoretical claims such as uniqueness of the representations throughout the network, and their stable estimation, all guaranteed under simple local sparsity conditions. Lastly, identifying the weaknesses in the above scheme, we propose an alternative to the forward-pass algorithm, which is both tightly connected to deconvolutional and recurrent neural networks, and has better theoretical guarantees.