On singularity properties of convolution of algebraic morphisms

Abstract:

In analysis, a convolution of two functions usually results in a smoother, better behaved function. Given two morphisms \( f, g \) from algebraic varieties \( X, Y \) to an algebraic group \( G \), one can define a notion of convolution of these morphisms. Analogously to the analytic situation, this operation yields a morphism (from \( X \times Y \) to \( G \)) with improved smoothness properties.

In this talk, I will define a convolution operation and discuss some of its properties. I will then present a recent result; if \( G \) is an algebraic group, \( X \) is smooth and absolutely irreducible, and \( f: X \to G \) is a dominant map, then after finitely many self convolutions of \( f \), we obtain a morphism with the property of being flat with fibers of rational singularities (a property which we call (FRS)).

Moreover, Aizenbud and Avni showed that the (FRS) property has an equivalent analytic characterization, which leads to various applications such as counting points of schemes over finite rings, representation growth of certain compact \( p \)-adic groups and arithmetic groups of higher rank, and random walks on (algebraic families of) finite groups. We will discuss some of these applications, and maybe some of the main ideas of the proof of the above result.

Joint with Yotam Hendel.