Singularity properties of convolutions of algebraic morphisms and probabilistic Waring type problems

Abstract:

Let $G$ be a connected algebraic group. We define and study a convolution operation between algebraic morphisms into $G$. We show that this operation yields morphisms with improved singularity properties, and in particular, that under reasonable assumptions one can always obtain a flat morphism with reduced fibers of rational singularities (termed an FRS morphism) after enough convolutions.

The FRS property is of high importance since (FRS) morphisms can be characterized by good asymptotic behaviour of the number of points of their fibers over finite rings of the form $\mathbb{Z}/p^k\mathbb{Z}$. This further allows us to interpret the FRS property through probabilistic lenses.

We discuss some of the above, motivated by the special case of word maps which can be viewed as a relative analogue in the settings of $p$-adic groups of Waring's problem from 1770 (see arXiv:1912.12556).

Joint work with Itay Glazer.