Some applications of the \( p \)-biased measure to Erdős-Ko-Rado type problems

Abstract:

If \( X \) is a finite set, the \( p \)-biased measure on the power-set of \( X \) is defined as follows: choose a subset \( S \) of \( X \) at random by including each element of \( X \) independently with probability \( p \). If \( \mathcal{F} \) is a family of subsets of \( X \), one can consider the \( \text{\em \( p \)-biased measure} \) of \( \mathcal{F} \), denoted by \( \mu_p(\mathcal{F}) \), as a function of \( p \); if \( \mathcal{F} \) is closed under taking supersets, then this function is an increasing function of \( p \). Seminal results of Friedgut and Friedgut-Kalai give criteria for this function to have a 'sharp threshold'. A careful analysis of the behaviour of this function also yields some rather strong results in extremal combinatorics which do not explicitly mention the \( p \)-biased measure - in particular, in the field of \( \text{\em Erdős-Ko-Rado type problems} \), which concern the sizes of families of objects in which any two (or three...) of the objects 'agree' or 'intersect' in some way. We will discuss some of these, including a recent proof of an old conjecture of Frankl that a symmetric three-wise intersecting family of subsets of \( \{1,2,\ldots,n\} \) has size \( o(2^n) \), and some 'stability' results characterizing the structure of 'large' \( t \)-intersecting families of \( k \)-element subsets of \( \{1,2,\ldots,n\} \). Based on joint work with (subsets of) Nathan Keller, Noam Lifshitz and Bhargav Narayanan.