Static Data Structure Lower Bounds Imply Rigidity

Abstract:

We show that static data structure lower bounds in the group (linear) model imply semi-explicit lower bounds on matrix rigidity. In particular, we prove that an explicit lower bound of \( t \gg \log^2(n) \) on the cell-probe complexity of linear data structures in the group model, even against arbitrarily small linear space \( (s = (1+\epsilon)n) \), would already imply a semi-explicit \( (P \wedge NP) \) construction of rigid matrices with significantly better parameters than the current state of art (Alon, Panigrahy, and Yekhanin, 2009). Our result further asserts that polynomial \( (t > n^{\epsilon}) \) data structure lower bounds against near-maximal space, would imply super-linear circuit lower bounds for log-depth linear circuits (a four-decade open question). In the succinct space regime \( (s = n+o(n)) \), we show that any improvement on current cell-probe lower bounds in the linear model would also imply new rigidity bounds. Our main result relies on a new connection between the "inner" and "outer" dimensions of a matrix (Paturi and Pudlak, 2006), and on a new worst-to-average case reduction for rigidity, which is of independent interest.

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