Static Data Structure Lower Bounds Imply Rigidity.

Abstract:

We show that static data structure lower bounds in the group (linear) model imply semi-explicit lower bounds on matrix rigidity. In particular, we prove that an explicit lower bound of \( t \geq \omega(\log^2 n) \) on the cell-probe complexity of linear data structures in the group model, even against arbitrarily small linear space \((s = (1+\epsilon)n)\), would already imply a semi-explicit \((P^NP)\) construction of rigid matrices with significantly better parameters than the current state of art (Alon, Panigrahy, and Yekhanin, 2009). Our result further asserts that polynomial \((t \geq n^\delta)\) data structure lower bounds against near-optimal space, would imply super-linear circuit lower bounds for log-depth linear circuits (which would close a four-decade open question). In the succinct space regime \((s = n+o(n))\), we show that any improvement on current cell-probe lower bounds in the linear model would also imply new rigidity bounds. Our main result relies on a new connection between the \(\text{inner} - \text{outer}\) dimensions of a matrix (Paturi and Pudlak, 2006), and on a new worst-to-average case reduction for rigidity, which is of independent interest.

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