Static Data Structure Lower Bounds Imply Rigidity

Abstract:

We show that static data structure lower bounds in the group (linear) model imply semi-explicit lower bounds on matrix rigidity. In particular, we prove that an explicit lower bound of \( t \gg \log^2(n) \) on the cell-probe complexity of linear data structures in the group model, even against arbitrarily small linear space \( (s = (1+\epsilon)n) \), would already imply a semi-explicit \( (P^{\text{NP}}) \) construction of rigid matrices with significantly better parameters than the current state of art (Alon, Panigrahy, and Yekhanin, 2009). Our result further asserts that polynomial \( (t > n^{\epsilon}) \) data structure lower bounds against near-maximal space, would imply super-linear circuit lower bounds for log-depth linear circuits (a four-decade open question). In the succinct space regime \( (s = n+o(n)) \), we show that any improvement on current cell-probe lower bounds in the linear model would also imply new rigidity bounds. Our main result relies on a new connection between the "inner" and "outer" dimensions of a matrix (Paturi and Pudlak, 2006), and on a new worst-to-average case reduction for rigidity, which is of independent interest.

Joint work with Zeev Dvir (Princeton) and Alexander Golovnev (Harvard).