Supersingular representations and the mod $p$ Langlands

Abstract:

Let $F/\mathbb{Q}_p$ be a finite extension, supersingular representations are the irreducible mod $p$ representations of $\text{GL}_n(F)$ which do not appear as a subquotient of a principal series representation, and similarly to the complex case, they are the building blocks of the representation theory of $\text{GL}_n(F)$. Historically, they were first discovered by L. Barthel and R. Livne some twenty years ago and they are still not understood even for $n=2$.

For $F=\mathbb{Q}_p$, the supersingular representations of $\text{GL}_2(F)$ have been classified by C. Breuil, and a local mod $p$ Langlands correspondence was established between them and certain mod $p$ Galois representations.

When one tries to generalize this connection and move to a non-trivial extension of $\mathbb{Q}_p$, Breuil's method fails; The supersingular representations in that case have complicated structure and instead of two as in the case $F=\mathbb{Q}_p$ we get infinitely many such representations, when there are essentially only finitely many on the Galois side.

In this talk we give an exposition of the subject and explore, using what survives from Breuil's methods, the universal modules whose quotients contain all the supersingular representations in the difficult case where $F$ is a non-trivial extension of $\mathbb{Q}_p$. 