Tightness for the Cover Time of $S^2$

Abstract:

Let $M$ be a smooth, compact, connected two-dimensional, Riemannian manifold without boundary, and let $C_{\epsilon}$ be the amount of time needed for the Brownian motion to come within (Riemannian) distance $\epsilon$ of all points in $M$. The first order asymptotics of $C_{\epsilon}$ as $\epsilon$ goes to 0 are known. We show that for the two dimensional sphere

$$\sqrt{C_{\epsilon}} - 2\sqrt{2} \left( \log \frac{1}{\epsilon} - \frac{1}{4} \log \log \frac{1}{\epsilon} \right)$$

is tight.

Joint work with David Belius and Ofer Zeitouni.