Title: 2,3,…,k: From approximating the number of edges to approximating the number of k-cliques (with a sublinear number of queries)

Abstract:

In this talk I will present an algorithms for approximating the number of k-cliques in a graph when given query access to the graph. This problem was previously studied for the cases of k=2 (edges) and k=3 (triangles). We give an algorithm that works for any $k \geq 3$, and is actually conceptually simpler than the $k=3$ algorithm. We consider the standard query model for general graphs via (1) degree queries, (2) neighbor queries and (3) pair queries. Let $n$ denote the number of vertices in the graph, $m$ the number of edges, and $C_k$ the number of k-cliques. We design an algorithm that outputs a $(1+\epsilon)$-approximation (with high probability) for $C_k$, whose expected query complexity and running time are $O(\frac{n}{C_k^{1/k}}+\frac{m^{k/2}}{C_k}) \text{poly} (\log n, \frac{1}{\epsilon}, k)$.

Hence, the complexity of the algorithm is sublinear in the size of the graph for $C_k = \Omega(m^{k/2-1})$. Furthermore, we prove a lower bound showing that the query complexity of our algorithm is essentially optimal (up to the dependence on $\log n, \frac{1}{\epsilon}$ and $k$).

This is joint work with Talya Eden and C. Seshadhri.