Towards the global bifurcation theory on the plane

Abstract:

The talk provides a new perspective of the global bifurcation theory on the plane. Theory of planar bifurcations consists of three parts: local, nonlocal and global ones. It is now clear that the latter one is yet to be created. Local bifurcation theory (in what follows we will talk about the plane only) is related to transfigurations of phase portraits of differential equations near their singular points. This theory is almost completed, though recently new open problems occurred. Nonlocal theory is related to bifurcations of separatrix polygons (polycycles). Though in the last 30 years there were obtained many new results, this theory is far from being completed. Recently it was discovered that nonlocal theory contains another substantial part: a global theory. New phenomena are related with appearance of the so called sparkling saddle connections. The aim of the talk is to give an outline of the new theory and discuss numerous open problems. The main new results are: existence of an open set of structurally unstable families of planar vector fields, and of families having functional invariants (joint results with Kudryashov and Schurov). Thirty years ago Arnold stated six conjectures that outlined the future development of the global bifurcation theory in the plane. All these conjectures are now disproved. Though the theory develops in quite a different direction, this development is motivated by the Arnold's conjectures.