Abstract:

Root-reductive Lie algebras form a special type of reasonably well behaved infinite-dimensional Lie algebras. In this talk, we shall define a version of Bernstein-Gelfand-Gelfand categories $O$ for root-reductive Lie algebras, which we called extended categories $O$ and briefly discuss some properties of these categories. Let $g$ be a root-reductive Lie algebra containing a splitting Borel subalgebra $b$ satisfying a special additional condition called the Dynkin condition. The extended category $O$ corresponding to $g$ and $b$ is denoted by $O$-$\text{bar}$.

The category $O$-$\text{bar}$ can be decomposed analogously to the finite-dimensional cases into blocks. The main object of this talk is to give a construction of translation functors of $O$-$\text{bar}$. Then we shall see that some objects such as tilting modules arise by applying the translation functors to Verma modules just as in the finite-dimensional cases. Furthermore, the translation functors establish equivalences between some blocks of the category $O$-$\text{bar}$. 