Translation Functors of Categories $\mathcal{O}$ for Root-Reductive Lie Algebras

Abstract:

Root-reductive Lie algebras form a special type of reasonably well behaved infinite-dimensional Lie algebras. In this talk, we shall define a version of Bernstein-Gelfand-Gelfand categories $\mathcal{O}$ for root-reductive Lie algebras, which we called extended categories $\mathcal{O}$ and briefly discuss some properties of these categories. Let $g$ be a root-reductive Lie algebra containing a splitting Borel subalgebra $b$ satisfying a special additional condition called the Dynkin condition. The extended category $\mathcal{O}$ corresponding to $g$ and $b$ is denoted by $\mathcal{O}\text{-bar}$.

The category $\mathcal{O}\text{-bar}$ can be decomposed analogously to the finite-dimensional cases into blocks. The main object of this talk is to give a construction of translation functors of $\mathcal{O}\text{-bar}$. Then we shall see that some objects such as tilting modules arise by applying the translation functors to Verma modules just as in the finite-dimensional cases. Furthermore, the translation functors establish equivalences between some blocks of the category $\mathcal{O}\text{-bar}$. 