Translation Functors of Categories O for Root-Reductive Lie Algebras

Abstract:

Root-reductive Lie algebras form a special type of reasonably well behaved infinite-dimensional Lie algebras. In this talk, we shall define a version of Bernstein-Gelfand-Gelfand categories \( O \) for root-reductive Lie algebras, which we called extended categories \( O \) and briefly discuss some properties of these categories. Let \( g \) be a root-reductive Lie algebra containing a splitting Borel subalgebra \( b \) satisfying a special additional condition called the Dynkin condition. The extended category \( O \) corresponding to \( g \) and \( b \) is denoted by \( O\text{-bar} \).

The category \( O\text{-bar} \) can be decomposed analogously to the finite-dimensional cases into blocks. The main object of this talk is to give a construction of translation functors of \( O\text{-bar} \). Then we shall see that some objects such as tilting modules arise by applying the translation functors to Verma modules just as in the finite-dimensional cases. Furthermore, the translation functors establish equivalences between some blocks of the category \( O\text{-bar} \).