Trees for Vertex Cuts, Hypergraph Cuts and Minimum Hypergraph Bisection

Abstract:

In the Minimum Hypergraph Bisection problem, the vertex set of a hypergraph has to be partitioned into two parts of equal size so that the number of hyperedges intersecting both parts is minimized. This problem is a natural generalization of the well-studied Minimum Bisection problem in graphs. We present a sharp distinction between Minimum Bisection in hypergraphs and graphs. Whereas it is well-known that all bi-criteria approximation algorithms for Minimum Bisection in graphs can be extended to hypergraphs with the exact same guarantees, we prove that this is not the case when considering true (i.e., non bi-criteria) approximation algorithms. Specifically, we show that Minimum Hypergraph Bisection admits an $\tilde{\mathcal{O}}(\sqrt{n})$ approximation algorithm. However, we also show that any $\alpha$-approximation algorithm for Minimum Hypergraph Bisection implies an approximation of $\Omega(\alpha^{-2})$ for Densest $k$-Subgraph. Thus, assuming the exponential time hypothesis there is no efficient approximation algorithm for Minimum Hypergraph Bisection with an approximation ratio $n^{\text{poly}(-\log(-\log(n)))}$. In particular, Minimum Hypergraph Bisection is much harder to approximate than Minimum Bisection in graphs, for which a logarithmic approximation algorithm exists. If time permits, the problem of constructing trees that are cut sparsifiers for hypergraph and vertex cuts will also be discussed. While similar trees lie at the heart of powerful algorithms for Minimum Bisection in graphs, we prove that this is not the case for hypergraphs. Joint work with Harald R"{a}cke and Richard Stotz.