Two-sided radial SLE and length-biased chordal SLE

Abstract:

Models in statistical physics often give measures on self-avoiding paths. We can restrict such a measure to the paths that pass through a marked point, obtaining a “pinned measure”. The aggregate of the pinned measures over all possible marked points is just the original measure biased by the path’s length. Does the analogous result hold for SLE curves, which appear in the scaling limits of many such models at criticality? We show that it does: the aggregate of two-sided radial SLE is length-biased chordal SLE, where the path’s length is measured in the natural parametrisation.