Typically-Correct Derandomization for Small Time and Space

Suppose a language $L$ can be decided by a bounded-error randomized algorithm that runs in space $S$ and time $n \cdot \text{poly}(S)$. We give a randomized algorithm for $L$ that still runs in space $O(S)$ and time $n \cdot \text{poly}(S)$ that uses only $O(S)$ random bits; our algorithm has a low failure probability on all but a negligible fraction of inputs of each length. An immediate corollary is a deterministic algorithm for $L$ that runs in space $O(S)$ and succeeds on all but a negligible fraction of inputs of each length. We also discuss additional complexity-theoretic applications of our technique.