Units, \( K \)-theory, and knots

Abstract:

In knot theory and more generally the theory of 3-manifolds various "quantum invariants" like the Witten-Reshetikhin-Turaev or the Kashaev invariant have been much studied in recent years, in particular because of the famous "volume conjecture" related to the asymptotic growth of the Kashaev invariant. Rather surprisingly, it transpired a few years ago that these invariants also have very non-trivial number-theoretical properties, including a kind of weak invariance under the modular group \( \text{SL}(2,\mathbb{Z}) \) ("quantum modular forms") and the experimental discovery of the appearance of certain units in cyclotomic extensions as factors in the asymptotic expansions. The talk will report on this and specifically on recent joint work with Frank Calegari and Stavros Garoufalidis that constructs such units in a purely algebraic way starting from elements in algebraic \( K \)-theory or in the more elementary "Bloch group". As an unexpected application, this result led to the proof of a well-known conjecture of Nahm on the modularity of certain \( q \)-hypergeometric series.