Abstract:

In knot theory and more generally the theory of 3-manifolds various "quantum invariants" like the
Witten-Reshetikhin-Turaev or the Kashaev invariant have been much studied in recent years, in
particular because of the famous "volume conjecture" related to the asymptotic growth of the
Kashaev invariant. Rather surprisingly, it transpired a few years ago that these invariants also have
very non-trivial number-theoretical properties, including a kind of weak invariance under the modular
group SL(2,Z) ("quantum modular forms") and the experimental discovery of the appearance of
certain units in cyclotomic extensions as factors in the asymptotic expansions. The talk will report on
this and specifically on recent joint work with Frank Calegari and Stavros Garoufalidis that constructs
such units in a purely algebraic way starting from elements in algebraic K-theory or in the more
elementary "Bloch group". As an unexpected application, this result led to the proof of a well-known
conjecture of Nahm on the modularity of certain q-hypergeometric series.