מכון ויצמן למדע

THE WEIZMANN INSTITUTE OF SCIENCE

FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Geometric Functional Analysis and Probability Seminar

Room 290C ,Ziskind Building
on Thursday, Feb 09, 2017
at 11:15

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Sydney

The values of quadratic forms on difference sets, measure rigidity and equidistribution

Abstract:

Given a quadratic form Q in d variables over the integers, e.g. $Q(x,y,z) = xy - z^2$, and a set of positive density E in Z^d, we investigate what kind of structure can be found in the set Q(E-E).
We will see that if $d \geq 3$, and Q is indefinite, then the measure rigidity, due to Bourgain-Furman-Lindenstrauss-Mozes or Benoist-Quint, of the action of the group of the symmetries of Q implies that there exists $k \geq 1$ such that $k^2 * Q(Z^d)$ is a subset of Q(E-E).
We will give an alternative proof of the theorem for the case $Q(x,y,z) = xy - z^2$ that uses more classical equidistribution results of Vinogradov, and Weyl, as well as a more recent result by Frantzikinakis-Kra. The latter proof extends the theorem to other polynomials having a much smaller group of symmetries. Based on joint works with M. Bjorklund (Chalmers), and K. Bulinski (Sydney).