Voter model percolation

Abstract:

The voter model on $\mathbb{Z}^d$ is a particle system that serves as a rough model for changes of opinions among social agents or, alternatively, competition between biological species occupying space. When $d \geq 3$, the set of (extremal) stationary distributions is a family of measures $\mu_\alpha$, for $\alpha$ between 0 and 1. A configuration sampled from $\mu_\alpha$ is a strongly correlated field of 0’s and 1’s on $\mathbb{Z}^d$ in which the density of 1’s is $\alpha$.

We consider such a configuration as a site percolation model on $\mathbb{Z}^d$. We prove that if $d \geq 5$, the probability of existence of an infinite percolation cluster of 1’s exhibits a phase transition in $\alpha$. If the voter model is allowed to have sufficiently spread-out interactions, we prove the same result for $d \geq 3$.

These results partially settle a conjecture of Bricmont, Lebowitz and Maes (1987). Joint work with Daniel Valesin (University of Groningen)