Weyl group representations and Harish-Chandra cells

Abstract:

Suppose \( g \) is a semisimple Lie algebra with Weyl group \( W \). Write \( L(w) \) for the irreducible highest weight module of highest weight \(-\rho - \rho\). Write \( J \) (for "Joseph") for the set of primitive ideals in a semisimple enveloping algebra contained in the augmentation ideal. In a 1978 paper "\( W \)-module structure in the primitive spectrum..." Joseph attached to each primitive ideal \( I \) in \( J \) a subset \( Lcell(I) = \{ w \in W \mid \text{Ann}(L(w)) = I \} \). He showed also how to make \( Lcell(I) \) into a basis for a representation \( \sigma(I) \) of \( W \), in such a way that \( \sum_{I \in J} \sigma(I) = \text{regular representation of } W \). These representations \( \sigma(I) \) are now called "left cell representations," terminology that is apparently due to Joseph (see his 1981 paper "Goldie rank in the enveloping algebra...III," page 310). Joseph proved in a 1980 paper that each left cell representation consists of exactly one copy of Joseph's "Goldie rank representation" for the primitive ideal \( I \), and some additional representations that are not Goldie rank representations. For the past forty years, understanding of these left cell representations of \( W \) has been at the heart of a great deal of work on representations of reductive groups. Lusztig in his 1984 book gave a description of all left cells in terms of the geometry of nilpotent orbits. Part of Lusztig's description uses Springer's parametrization of \( W \) representations by irreducible representations of the equivariant fundamental group \( A(O) \) for a nilpotent orbit \( O \). I will discuss the "opposite" part of Lusztig's description, involving conjugacy classes in \( A(O) \).