What do algebras form? (Revisited)

Abstract:
We will start with the observation that associative algebras form a two-category with a trace functor where one-morphisms are bimodules, two-morphisms are bimodule homomorphisms, and the trace of an \((A,A)\) bimodule \(M\) is \(M/[M,A]\). We then explain in what sense the derived version of the above is true, i.e. what happens when one replaces bimodule homomorphisms and the trace by their derived functors that are Hochschild (co)homology. We will explain how the beginnings of noncommutative differential calculus can be deduced from the above. This is a continuation of a series of works of MacClure and Smith, Tamarkin, Lurie, and others, and a joint work with Rebecca Wei.