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What do algebras form? (Revisited)

Abstract:

We will start with the observation that associative algebras form a two-category with a trace functor  
where one-morphisms are bimodules, two-morphisms are bimodule homomorphisms, and the trace of  
an (A,A) bimodule M is M/[M,A]. We then explain in what sense the derived version of the above is  
true, i.e. what happens when one replaces bimodule homomorphisms and the trace by their derived  
functors that are Hochschild (co)homology. We will explain how the beginnings of noncommutative  
differential calculus can bee deduced from the above. This is a continuation of a series of works of  
MacClure and Smith, Tamarkin, Lurie, and others, and a joint work with Rebecca Wei.