Why gradient flows of some energies good for defect equilibria are not good for dynamics, and an improvement

Abstract:

Line defects appear in the microscopic structure of crystalline materials (e.g. metals) as well as liquid crystals, the latter an intermediate phase of matter between liquids and solids. Mathematically, their study is challenging since they correspond to topological singularities that result in blow-up of total energies of finite bodies when utilizing most commonly used classical models of energy density; as a consequence, formulating nonlinear dynamical models (especially pde) for the representation and motion of such defects is a challenge as well. I will discuss the development and implications of a single pde model intended to describe equilibrium states and dynamics of these defects. The model alleviates the nasty singularities mentioned above and it will also be shown that incorporating a conservation law for the topological charge of line defects allows for the correct prediction of some important features of defect dynamics that would not be possible just with the knowledge of an energy function.

This is joint work with Chiqun Zhang, Dmitry Golovaty, and Noel Walkington.