Wilkie's conjecture for restricted elementary functions

Abstract:
Let X be a set definable in some o-minimal structure. The Pila-Wilkie theorem (in its basic form) states that the number of rational points in the transcendental part of X grows sub-polynomially with the height of the points. The Wilkie conjecture stipulates that for sets definable in $R_{\exp}$, one can sharpen this asymptotic to polylogarithmic.

I will describe a complex-analytic approach to the proof of the Pila-Wilkie theorem for subanalytic sets. I will then discuss how this approach leads to a proof of the "restricted Wilkie conjecture", where we replace $R_{\exp}$ by the structure generated by the restrictions of $\exp$ and $\sin$ to the unit interval (both parts are joint work with Dmitry Novikov). If time permits I will discuss possible generalizations and applications.