Abstract:

Every word $w(x_1,\ldots, x_r)$ in a free group, such as the commutator word $w=xyx^{-1}y^{-1}$, induces a word map $w:G^r \to G$ on every group $G$. For $g$ in $G$, it is natural to ask whether the equation $w(x_1,\ldots, x_r)=g$ has a solution in $G^r$, and to estimate the "size" of this solution set, in a suitable sense. When $G$ is finite, or more generally a compact group, this becomes a probabilistic problem of analyzing the distribution of $w(x_1,\ldots, x_r)$, for Haar-random elements $x_1,\ldots, x_r$ in $G$. When $G$ is an algebraic group, such as $\text{SL}_n(C)$, one can study the geometry of the polynomial map $w:\text{SL}_n(C)^r \to \text{SL}_n(C)$, using algebraic methods.

Such problems have been studied in the last few decades, in various settings such as finite simple groups, compact $p$-adic groups, compact Lie groups, and simple algebraic groups. Analogous problems have been studied for Lie algebra word maps as well. In this talk, I will mention some of these results, and explain the tight connections between the probabilistic and algebraic approaches.

Based on joint works with Yotam Hendel, Raf Cluckers and Nir Avni.