Zero-Convex Functions, Perturbation Resilience, and Subgradient Projections for Feasibility-Seeking Methods

Abstract:
The convex feasibility problem (CFP) is at the core of the modeling of many problems in various areas of science. Subgradient projection methods are important tools for solving the CFP because they enable the use of subgradient calculations instead of orthogonal projections onto the individual sets of the problem. Working in a real Hilbert space, we show that the sequential subgradient projection method is perturbation resilient. By this we mean that under appropriate conditions the sequence generated by the method converges weakly, and sometimes also strongly, to a point in the intersection of the given subsets of the feasibility problem, despite certain perturbations which are allowed in each iterative step. Unlike previous works on solving the convex feasibility problem, the involved functions, which induce the feasibility problem’s subsets, need not be convex. Instead, we allow them to belong to a wider and richer class of functions satisfying a weaker condition, that we call “zero-convexity”. This class, which is introduced and discussed here, holds a promise to solve optimization problems in various areas, especially in non-smooth and non-convex optimization. The relevance of this study to approximate minimization and to the recent superiorization methodology for constrained optimization is explained.

This is a joint work with Yair Censor.